Green OFDM Transmission:

An Optimal Signal Design Technique

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- 10% of the world's energy consumption is due to the Information and Communications Technology (ICT) industry
- Energy-efficiency has become one of the key performance indicators (KPI) in the design and implementation of radio systems
- Green radio Communications is to design energy-efficient wireless communication techniques and protocols which optimally utilize available resources and minimize power consumption.

Various Strategies for Designing Energy-efficient Wireless Systems

- **Energy Efficient New Radios**
- **Minimization of Interference**
- **Optimal resource Allocation**

The method developed in this work falls in the category of Efficient New Radios.

Approach: An optimization problem

We would like to design an OFDM signal

with least transmit power and the **best performance**

in the **BPSK data detection**

OFDM Transmission

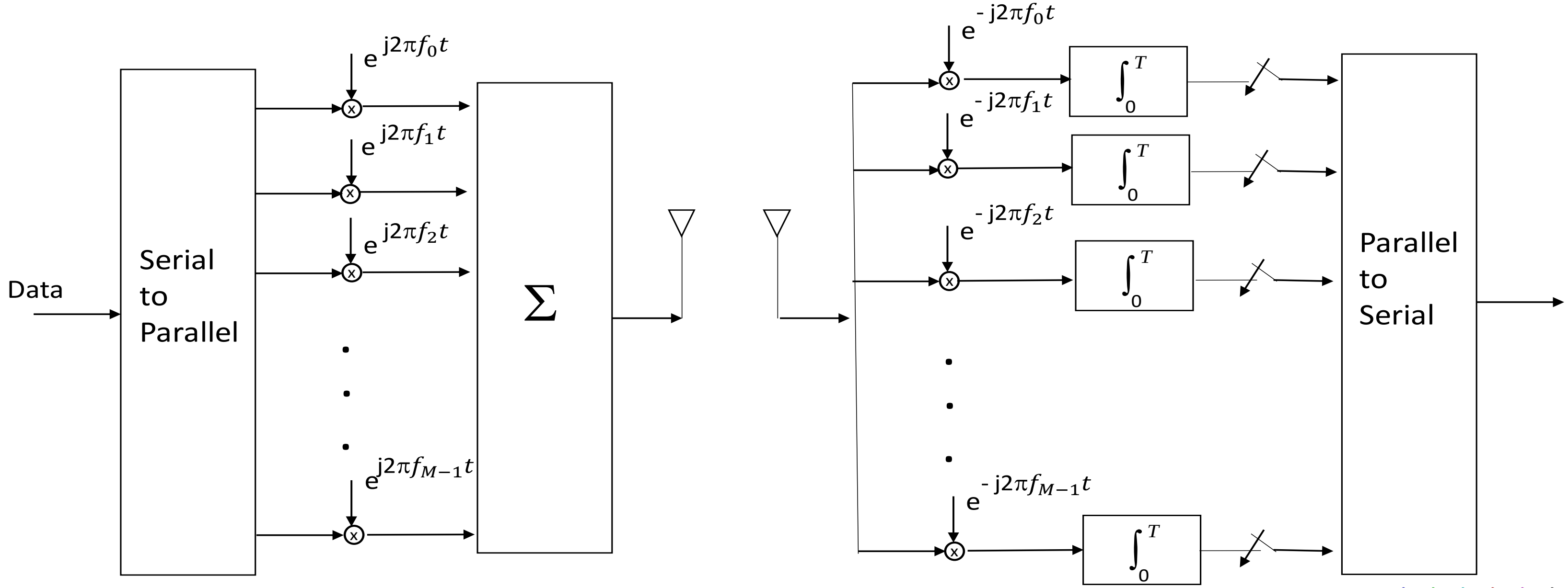
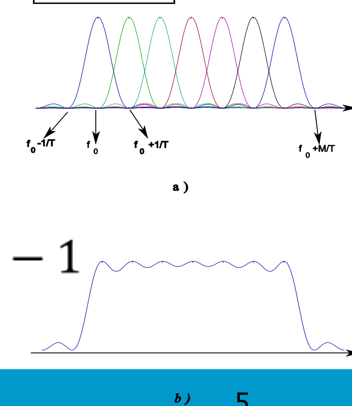


Fig. 1: Block diagram of OFDM system



frequency of m-th carrier is $\quad f_m = f_0 + \frac{m}{T} \qquad m = 0,1,2,\ldots,M-1$

M = number of sub-carriers, $\quad T$ = OFDM symbol duration

The OFDM transmitted signal is written as

$$s(t) = \frac{1}{\sqrt{M}} \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} b_m(i) e^{j2\pi f_m t} g(t - iT)$$

$b_m(i)$ = Symbol of the mth subchannel at time interval iT, and for the BPSK modulation is ± 1

$g(t)$ is the shape of the transmitter filter that is non-zero in $[0, T)$, which we will try to optimize its shape

The factor $1/\sqrt{M}$ is for keeping the power of the OFDM signal constant irrespective the number of subcarriers.

For the interval $[0, T)$ the OFDM signal can be expressed as:

$$s(t) = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} b_m(0) e^{j2\pi f_m t} g(t), \qquad 0 \le t < T$$

Since OFDM transmission is a very wideband transmission technique the transmit antenna will influence this signal.

The impact of the antenna in the transmission band is modeled as a *differentiator* .

Accordingly, the transmitted signal is written as:

$$x(t) = \frac{d}{dt} s(t)$$

$$= \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} b_m(0)[j2\pi f_m e^{j2\pi f_m t} g(t) + e^{j2\pi f}$$

$$0 \leq t < T$$

$$\dot{g}(t) = \frac{d}{dt} g(t)$$

Data detection Procedure in the OFDM Receiver

Ignoring noise at the receiver for the detection of the *k*th bit the following operation is done:

$$z_k = \int_0^T x(t) e^{-j2\pi f_k t} g(t) dt$$

$$= \int_0^T \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} b_m(0) e^{j2\pi f_m t} [j2\pi f_m g(t) + \dot{g}(t)] e^{-j2\pi f_k t} g(t) dt$$

The decision variable z_k can be written as:

$$z_k = \frac{1}{\sqrt{M}} \int_0^T b_k(0) [j2\pi f_k g(t) + \dot{g}(t)] g(t) dt + \frac{1}{\sqrt{M}} \sum_{m=0, m\neq k}^{M-1} b_m(0) e^{j2\pi (f_m - f_k) t} [j2\pi f_m g(t) + \dot{g}(t)]\, g(t) dt$$

useful signal for detection of bit b_k

inter-carrier interference (ICI)

For BPSK Modulation and ignoring ICI term the decision variable becomes:

$$z_k = \frac{1}{\sqrt{M}} \int_0^T b_k(0)[\dot{g}(t)]g(t)dt$$

So, for the best detection performance we would like to have

$$\int_0^T g(t)\dot{g}(t)dt = 1$$

This is a normalized constraint that will be used later in the optimization problem.

OFDM Signal Power

$$\boxed{x(t) = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} b_m(0) e^{j2\pi f_m t} [j2\pi f_m g(t) + \dot{g}(t)], \quad 0 \le t < T}$$

$$\text{Power} = |x(t)|^2$$
$$= \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} \sum_{n=0}^{M-1} b_m(0) b_n{}^*(0) e^{j2\pi (f_m - f_n)t} [j2\pi f_m g(t)$$
$$+ \dot{g}(t)][-j2\pi f_n g(t) + \dot{g}(t)]$$

the average power is obtained as:

$$\overline{|x(t)|^2} = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} \sum_{n=0}^{M-1} \overline{b_m(0) b_n{}^*(0)}\, e^{j2\pi (f_m - f_n)t} [j2\pi f_m g(t) + \dot{g}(t)][-j2\pi f_n g(t) + \dot{g}(t)]$$

Assuming independent $b_m(0)$ and $b_n{}^*(0)$

$$\overline{|x(t)|^2} = \frac{1}{M}\sum_{m=0}^{M-1} \overline{b_m{}^2(0)}\left[4\pi^2 f_m{}^2 g^2(t) + \dot{g}^2(t)\right]$$

$$\overline{|x(t)|^2} = A^2 g^2(t) + \dot{g}^2(t)$$

$$A^2 = \frac{2M(2M-1)\pi^2}{3T^2}$$

Optimization Problem

$$J_{min} = \int_0^T \overline{|x(t)|^2} dt = \int_0^T \left[A^2 g^2(t) + \dot{g}^2(t)\right] dt,$$

Subject to:

$$\int_0^T g(t)\dot{g}(t)dt = 1$$

Boundary conditions:

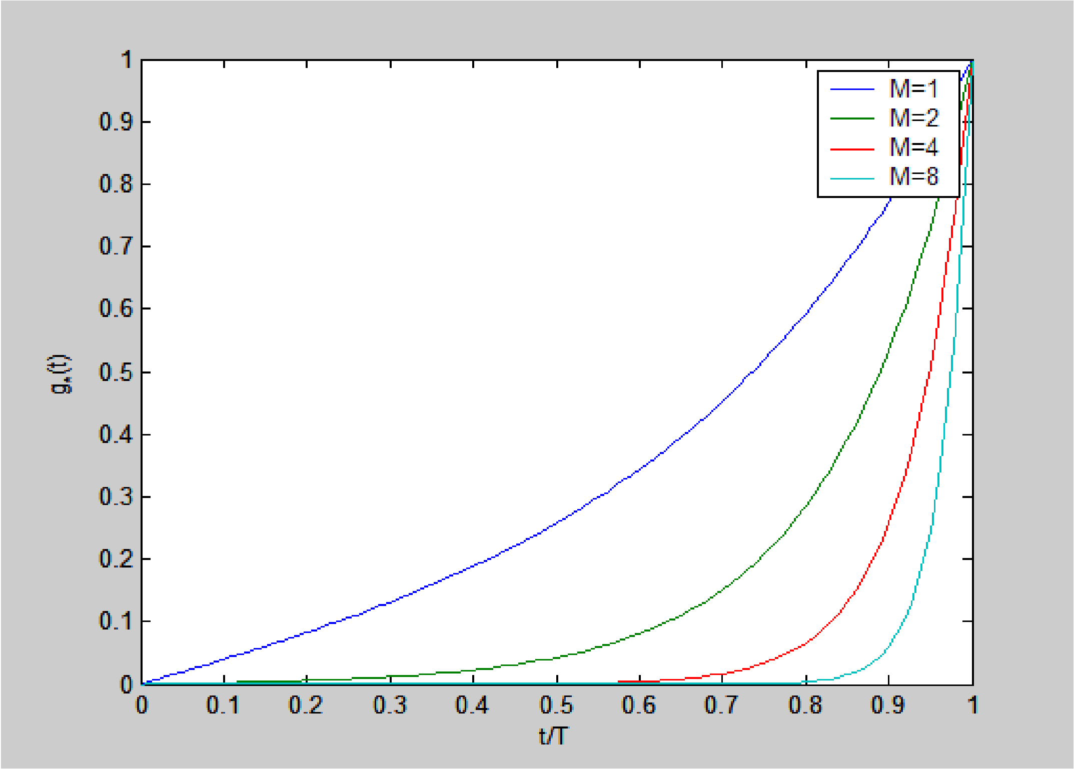
$g(0)=0$, and $g(T)=1$

For finding optimum waveform we use ***Calculus of Variations*** which leads to a set of differential equations (known as ***Euler equations***) with a *constraint* and *boundary conditions*.

Optimum waveform $g_*(t)$ for minimal energy becomes:

$$\boxed{g_*(t) = \frac{sinhAt}{sinh\,AT}}$$

$$A = \sqrt{\frac{2M(2M-1)\pi^2}{3T^2}}$$



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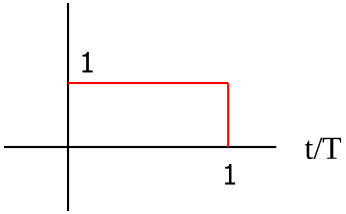
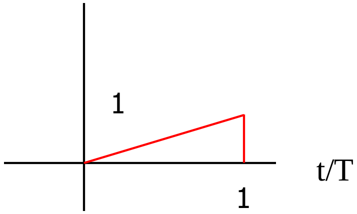
For comparison the normalized transmitted energy for the rectangular waveform $g_1(t)$:

$$g_1(t) = \begin{cases} 1 & 0 < \frac{t}{T} < 1 \\ 0 & otherwise \end{cases}$$



and the ramp waveform $g_2(t)$:

$$g_2(t) = \begin{cases} t_{/T} & 0 < \frac{t}{T} < 1 \\ 0 & otherwise \end{cases}$$



are compared with the optimal waveform $g_*(t)$

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Table1: **Normalized transmitted energy** for two typical and the optimum waveform

	M=2	M=4	M=8	M=16
rectangular g_1 (t)	1	1	1	1
ramp g_2 (t)	0.525	0.505	0.501	0.503
optimal g_* (t)	0.158	0.073	0.035	0.017

Conclusion

- We designed an ***optimal signal shape*** for energy **efficient OFDM transmission**.

- We started with the formulating of the **average power** of the **BPSK** modulated OFDM transmitted signal taking into account the behavior of the transmit antenna in the broad bandwidth of the OFDM signal.

- For the **least transmit power** and the **best performance** in the **BPSK data detection** and using the **calculus of variations** with the boundary conditions the optimal waveform was designed. Results show that the **sinus hyperbolic (*sinh*)-shaped** OFDM signal is the **optimal waveform**.

- The **design framework** presented in this work can directly be **applied** to other **modulation schemes** as well as **design criteria** (such as security, spectral efficiency, performance, etc.) of OFDM wireless communication networks by merely **changing the objective function**.

- Results show the **potential** of the technique to be further applied to smart **context-aware green OFDM wireless communications**.