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GDOP optimised LEO constellation for Positioning Estimation

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Objective

" To design a dedicated architecture of LEO constellation using GA which is aimed to minimize the GDOP and maximize the global coverage with at least 4 visible satellites at a given epoch."

Outline

- Review of the state of art in LEO PNT.
- Analysis of some selected constellations to get the initial hints to fix some orbital parameters.
- Mathematical modelling
- Genetic algorithm (GA) setting and implementation.
- Simulation, results and analysis.
- Conclusion and future steps.

State of Art in LEO PNT

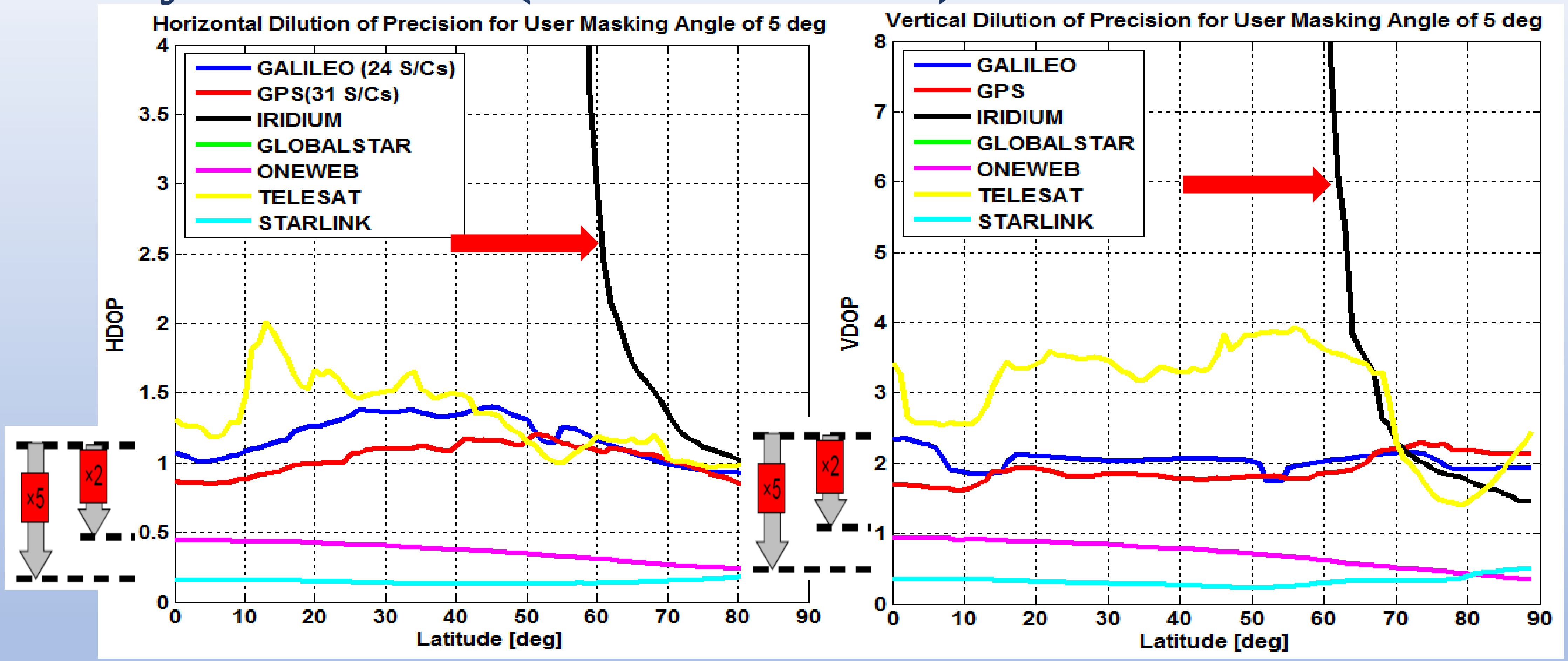
- **High velocity** →LEO **Doppler based positioning** → GNSS + Terrestrial Navigation.
- Lower altitude → free space losses **10 dB** lower w.r.t. MEO →LEO signals are more powerful than GNSS signals.
- Wide range of frequency bands available. **(UHF/VHF, L, S, C Ka, Ku etc)**
- Spreading loss at zenith for **LEO (-69dB), MEO (-97dB).**
- Reduce **multipath** in an urban canyon.
- **Fast convergence time (GNSS: 20-40 min, LEO: 1-10 min)**
- **More visible satellites for Mega constellations.**

Overview of the Orbital Parameters [1]

Constellation	Application	Altitude per orbit (Km)	Mean Velocity (Km/s)	Period (minutes)	Orbital Planes per Orbit	Satellites per Plane	Total Number of Satellites	Inclination (degrees)	Eccentricity	FrequencyBands// Downlink
GPS	Navigation	20200	3.88	720	6	4	24	55	0	L1 - 1575.42 Mhz L2 -1227.60 MHz
GALLILEO	Navigation	23222	3.66	845	3	6	24+6	56	0	E -1176-1207 MHz
StarLink	Global Internet Broadband	335.9	7.5576	91-112	9	77	11943	42	0	K-band:
		340.8	7.5839		7	354		48		17.8-18-5 GHz
		345.6	7.6098		9	283		53		18.8-19.3 GHz
		550	7.4984		24	66		53		19.7-20.2 GHz
		1110	7.2139		32	50		53.8		V-band:
		1130	7.2696		8	50		74		37.5-42.0 GHz
		1275	7.2128		5	75		81		
		1325	7.1679		6	75		70		
Iridium Next	Narrowband Communications	780	7.4628	97	6	11	66	86	0	L-band: 1.616-1.63 GHz K-band: 19.3-19.7 GHz
Oneweb	Global Internet Broadband	1200	7.24	110	18	36	358	87.9	0	Ku-Ka Bands
LEO - 192	Literature	1000	7,37	105	12		192	90	0	Not defined
		600	9.97	96	12			90		Not defined
LEO-288	Literature	1000	7,37	105	12		288	90		Not defined

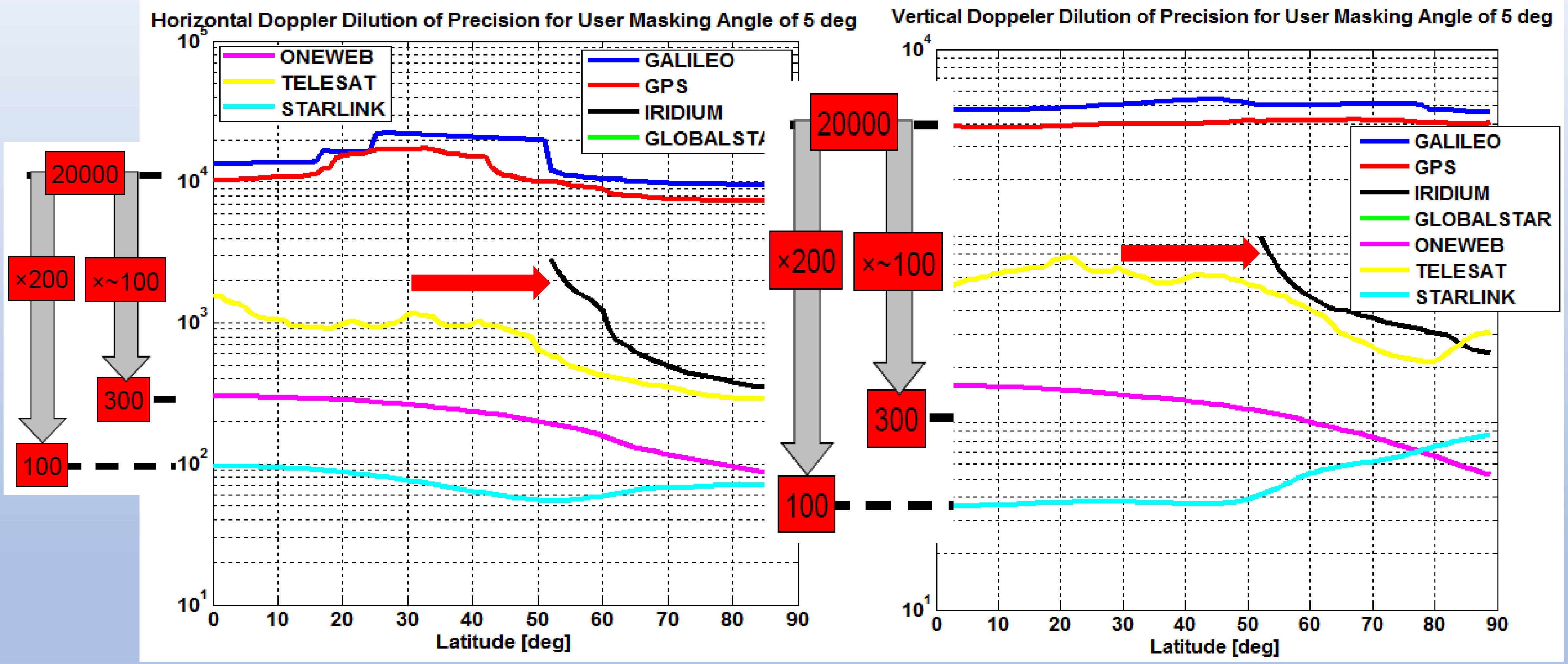
H = 1250 km, e = 0, w = 0, Hybrid Configuration (sub-Walker constellation)

Analysis of DOP (HDOP/VDOP) - I



- Starlink and Oneweb has 5X & 2X times less HDOP/VDOP with respect to Galileo and GPS.
- For Iridium minimum no. of visible satellites at a given epoch are lower than 4 so worst DOP [2].

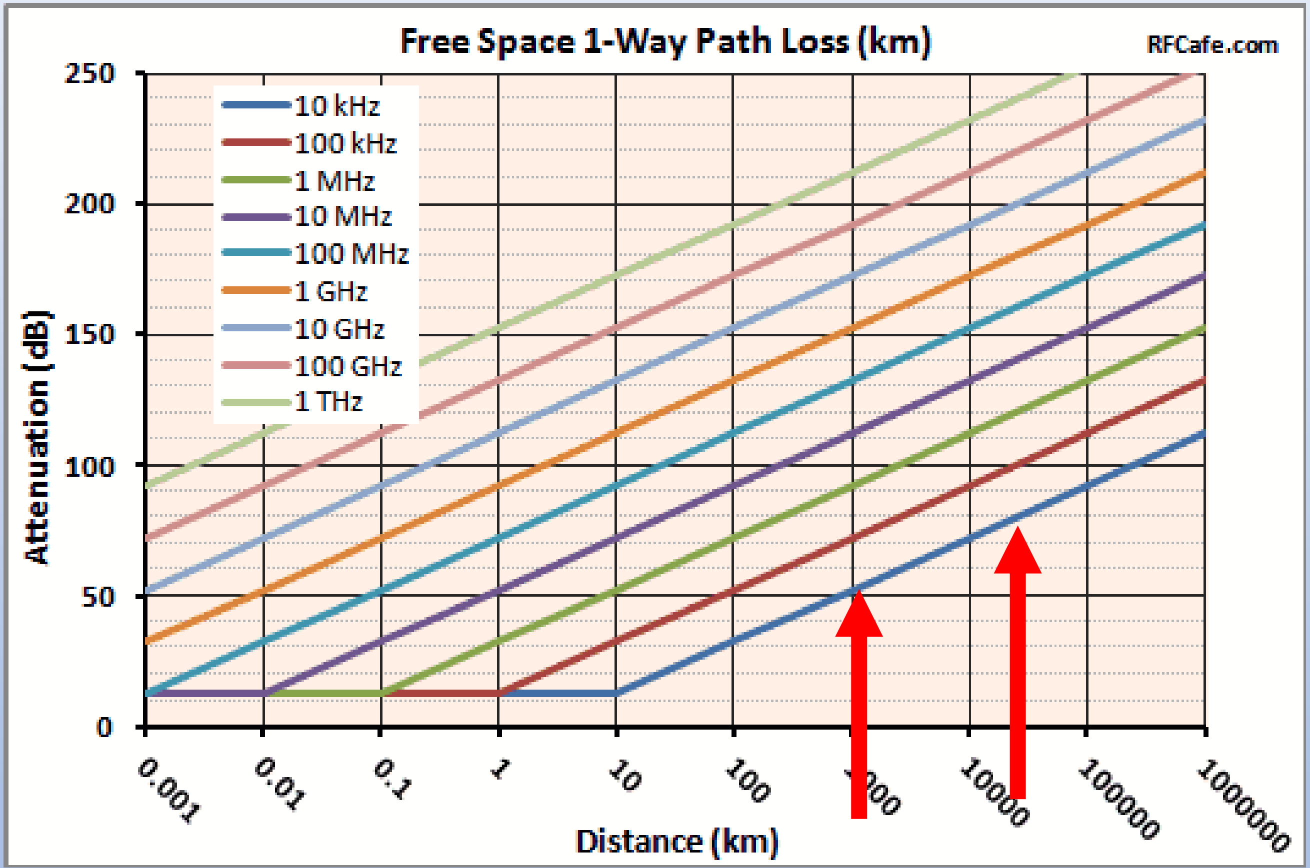
Analysis of Doppler DOP (HDDOP/VDDOP) - II



Mega Constellations DDOP is in order of 2 whereas, MEO in order of 4 as expected. (Results are verified in MATLAB [2])

Analysis of C/No and Free Space losses for different frequency bands - III

SNR is inversely proportional to path losses L_{FS}. From the formulation and table below free space losses for MEO (20000 km) and LEO (1000 km)



Variation attenuation against distance for different frequency range. Blue color box shows LEO and red color box shows MEO orbits [3].

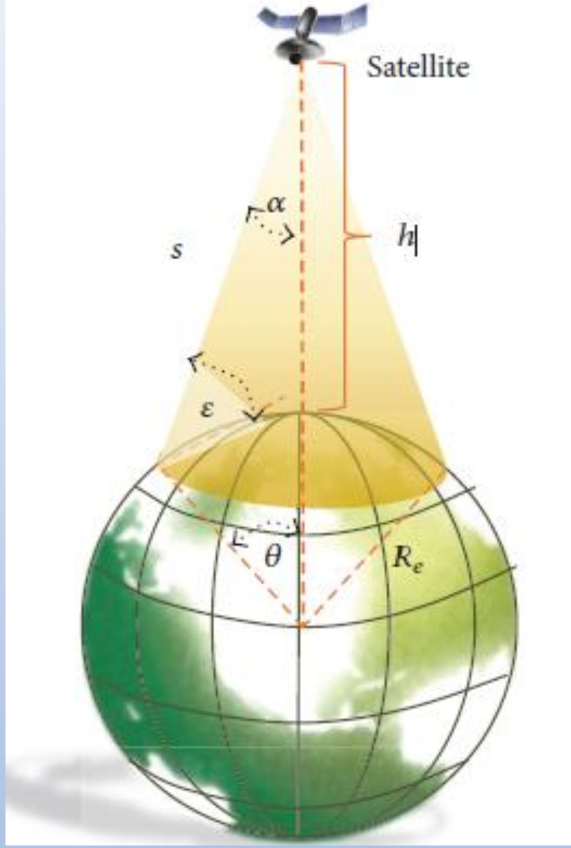
$$\left(\frac{C}{n_0}\right) = (EIRP\ \textit{Transmitter}) \times \left(\frac{1}{path\,loss}\right) \times \left(\frac{\textit{Satellite gain}}{\textit{Satellite noise temperature}}\right) \times \frac{1}{k}$$

$$L_{FS}(\mathrm{dB}) = 20\log_{10}\left(4\pi\frac{\text{distance}}{\text{wavelength}}\right)$$

Frequency bands	LEO (1000 Km)	MEO (20,000 km)
	Free space Losses in dB	
VHF	34.02	60.04
UHF	32.43	58.45
P	28	54.02
L	20.28	46.3
S	13.35	39.31
C	11.35	37.31
Ku	1.39	27.41
Ka	-2.97	23.04

Calculated free space losses using mentioned formulae. For given frequency bands. S, C bands shows three times less loss for LEO than MEO.

Mathematical Modelling



Single Satellite Coverage [4]

1. **Sub-Walker – I (Polar Orbits)**
2. **Equatorial Orthogonal Orbit**
3. **Sub-Walker – II (Optimized)**

Sub-Walker – I (Polar Orbits) & Equatorial Orthogonal Orbit

$$\Big\{(P_p - 1).\arcsin[\tan(\alpha)\cos(\pi/S_e)] + (P_p + 1)\arcsin\left[\frac{\sin(c)\cos(\pi/s_e)}{\cos(\alpha)}\right]\Big\}\eta = \pi$$

where;

$$c = \arccos\left[\frac{\cos(\alpha)}{\cos(pi/S_p)}\right]$$

Pp = no. of planes,
Sp = no. of satellites
Se = equatorial Sats
α = half cone angle,
η = * factor set to 1

Sub-Walker – II (a:I:T/P/F)

This is to be Optimised

$$\begin{cases} a_{ij} = a_0 \\ e_{ij} = e_0 \\ I_{ij} = I_0 \\ \Omega_{ij} = \Omega_0 + \frac{360^\circ}{P_w}\cdot(i-1) \\ \omega_{ij} = \omega_0 \\ \mathcal{M}_{ij} = \mathcal{M}_0 + \frac{360^\circ}{P_w S_w} F_w \cdot (i-1) + \frac{360^\circ}{S_w}\cdot(j-1) \end{cases}$$

a = semimajor axis,
I = inclination,
T = total number of satellite,
P =Total number of Planes (RAAN),
F - Phasing between (Mean Anomaly),
S = T*P

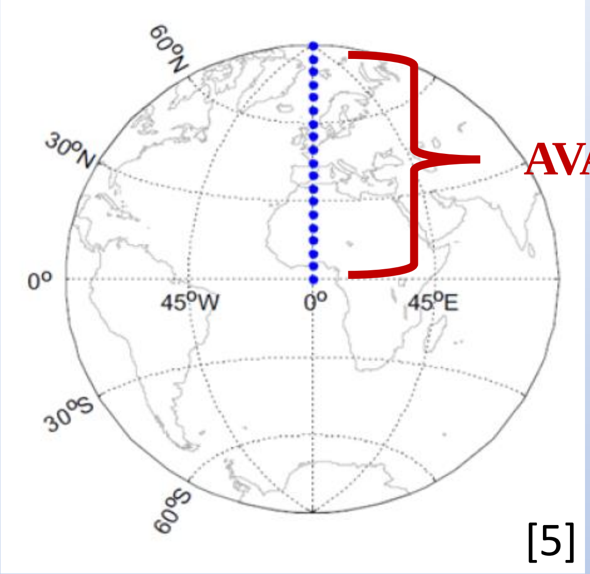
GA implementation to improve performance

1. GDOP - Minimise

GDOP	Rating
< 1	Ideal
1 < 2	Excellent
2 < 5	Good
5 < 10	Moderate

GOAL

2. Visibility of at least 4 satellites at global level (Maximise)



AVAILABILITY 100%

Overall Fitness Function

$$J = min\big((1-\beta).f_1(x) + \beta.(1 - f_2(x)\big), x \in C$$

Constraints

$$\begin{cases} \delta(\mathbf{x}) = 100\% \\ F_{\mathrm{w}} \leq P_{\mathrm{w}} - 1 \\ P_{\mathrm{w}} \in \mathbf{Z} \\ S_{\mathrm{w}} \in \mathbf{Z} \\ F_{\mathrm{w}} \in \mathbf{Z} \end{cases}$$

Optimization variables: X = [S, P, F, I]

GA settings an Hybrid Configurations

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Parameters	value
Population Size	300/400/500
Max generations	30/40/50/100
MINLP	S,P,F
Probability of crossover	0.8
Probability of mutation	0.194
Penalty	5

Setting of GA algorithm

X/ϵ	S1(7°)	S2(10°)	S3(15°)	S4(20°)	S5(20°)	S6(20°)
S_o	4-12	4-12	4-15	4-12	4-15	4-15
P_o	1-10	1-10	1-10	1-10	10-20	10-20
F_o	1-9	1-9	1-9	1-9	1-15	1-15
i_o	45°-60°	45°-60°	45°-60°	45°-60°	70°-90°	70°-90°

Search Ranges for optimization variables (X) for different configurations

Param	S1	S2	S3	S4	S5	S6
S_p	6	6	7	7	0	7
P_p	7	8	9	9	0	9
S_e	7	8	9	10	0	10
S_o	5	7	13	10	16	16
P_o	9	10	8	13	10	10
F_o	1	3	5	5	8	8
i_o	50.37°	46.58°	46.52°	50.50°	70.23°	70.23°
T_{sat}	94	120	176	203	160	233

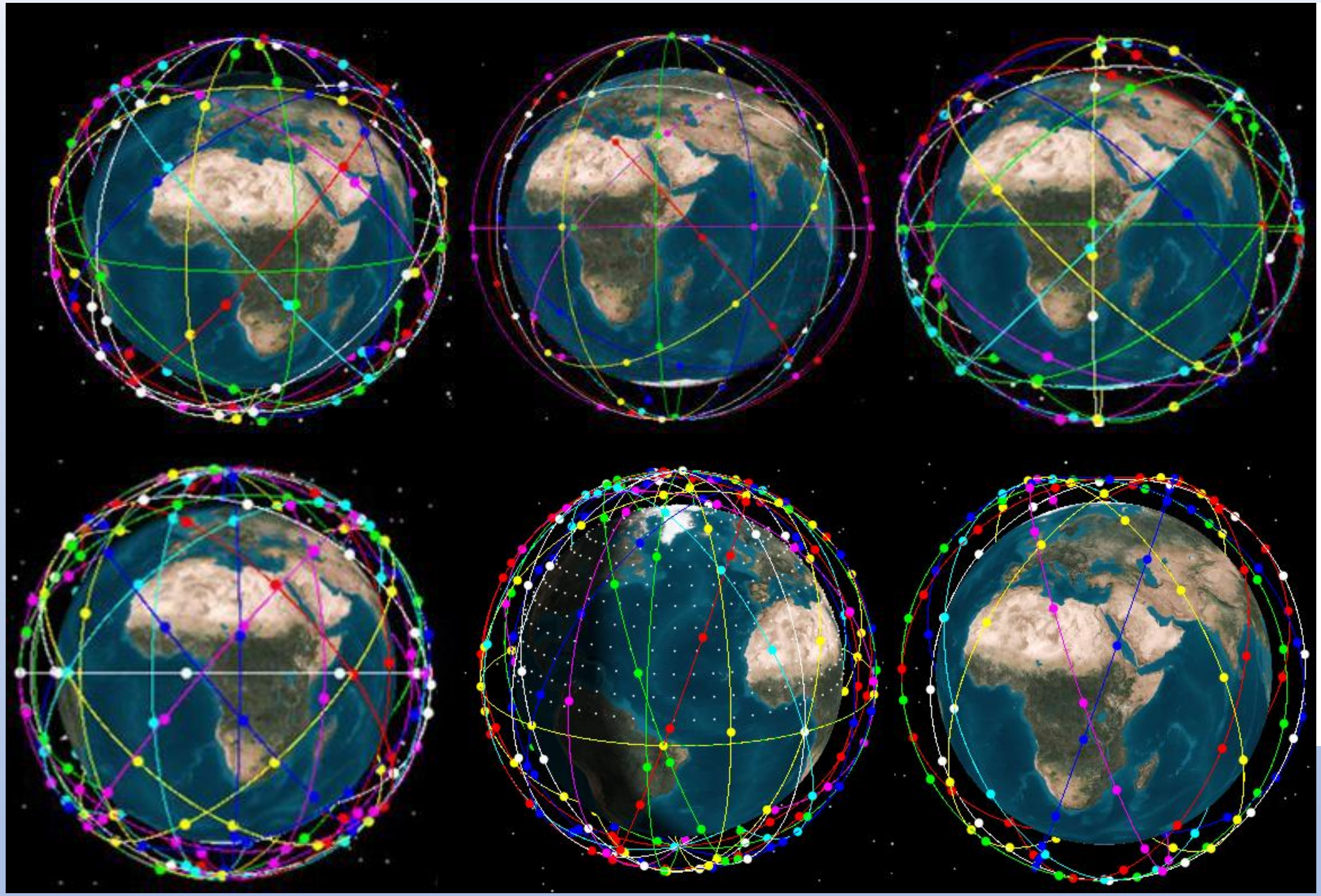
Polar Orbits

Equatorial Orbits

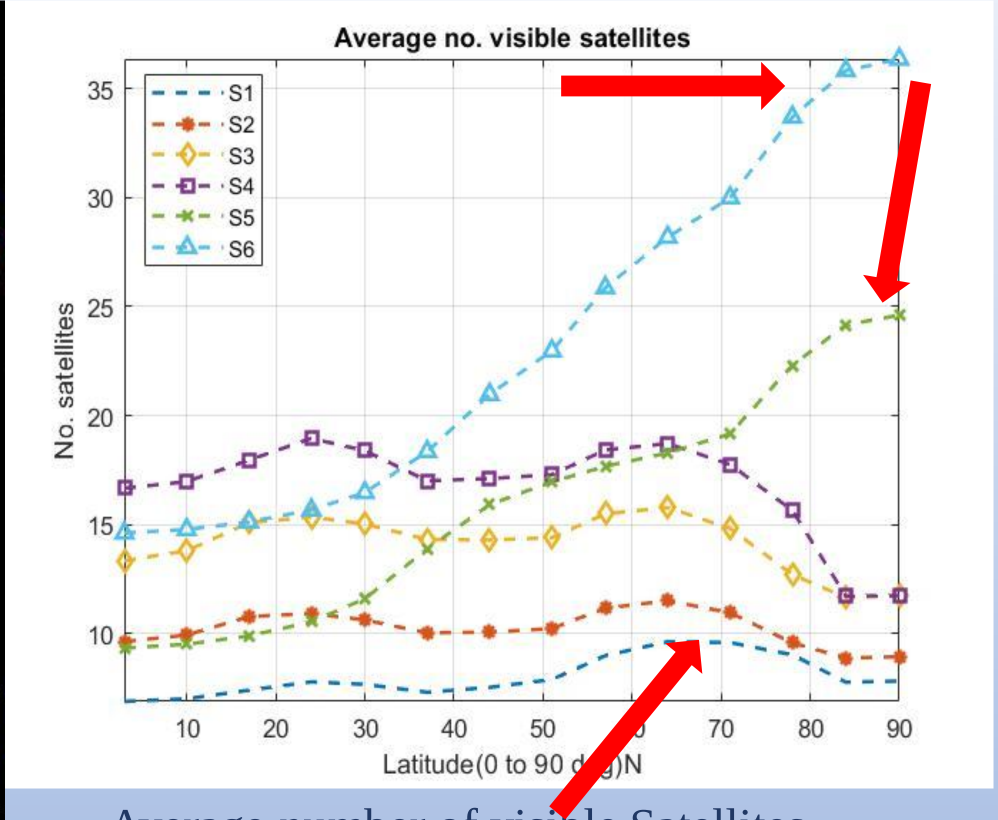
Optimized

Hybrid Configurations for different Elevation Mask angles

Results and Analysis – I (Visibility of Obtained configuration)

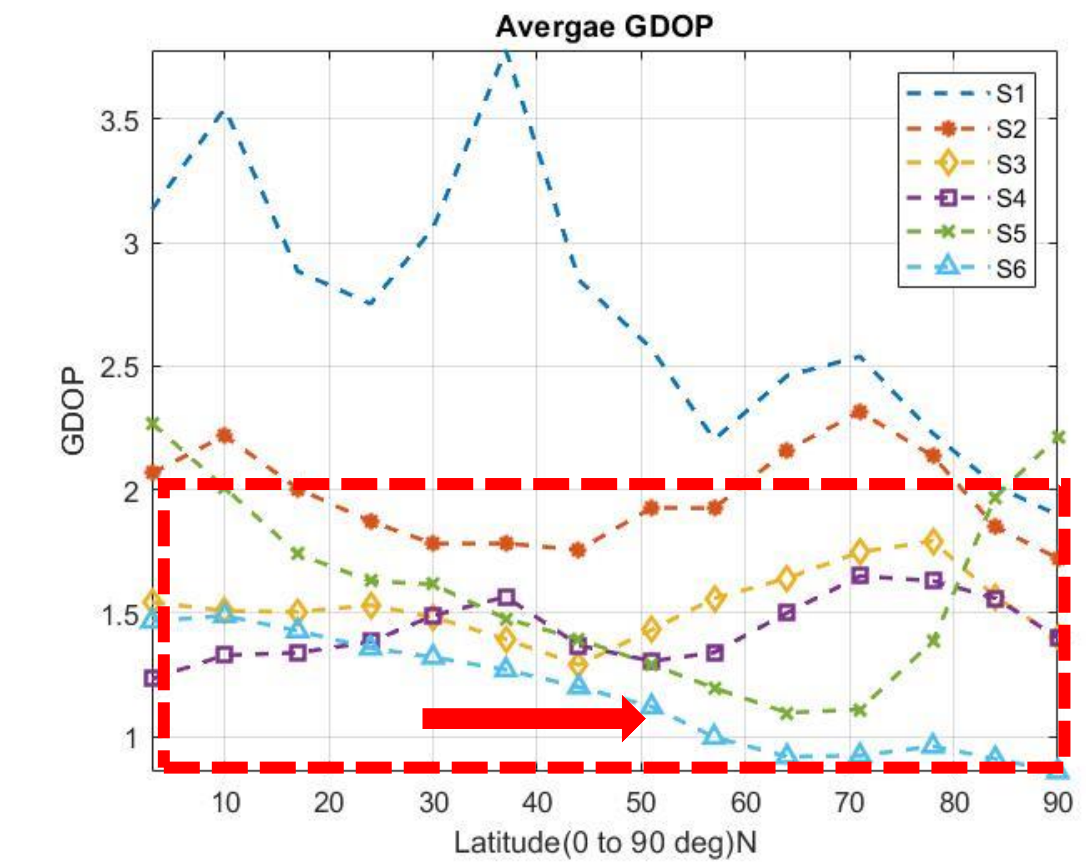


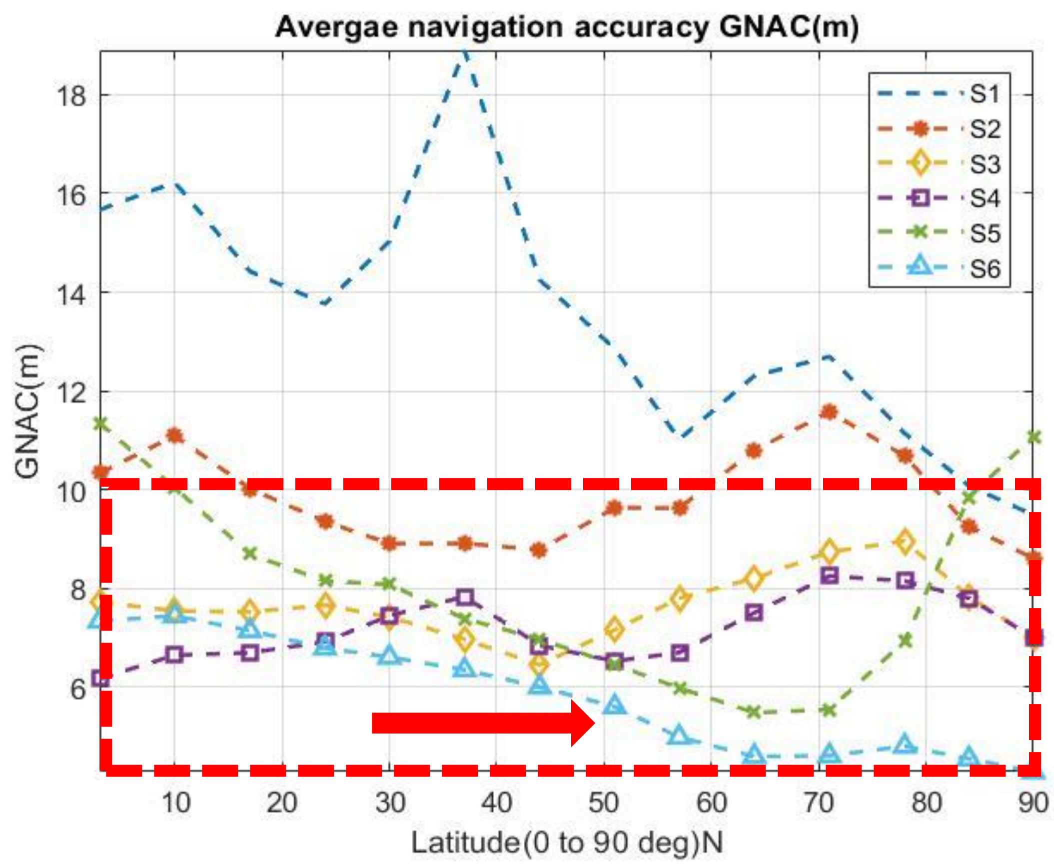
STK Plots for cases S1, S2, S3 (Top : L-R),
cases S4, S5, S6 (Bottom: L-R)



Average number of visible Satellites
over Latitude (0- 90 Deg)

Results and Analysis – II (GDOP and GNAC)





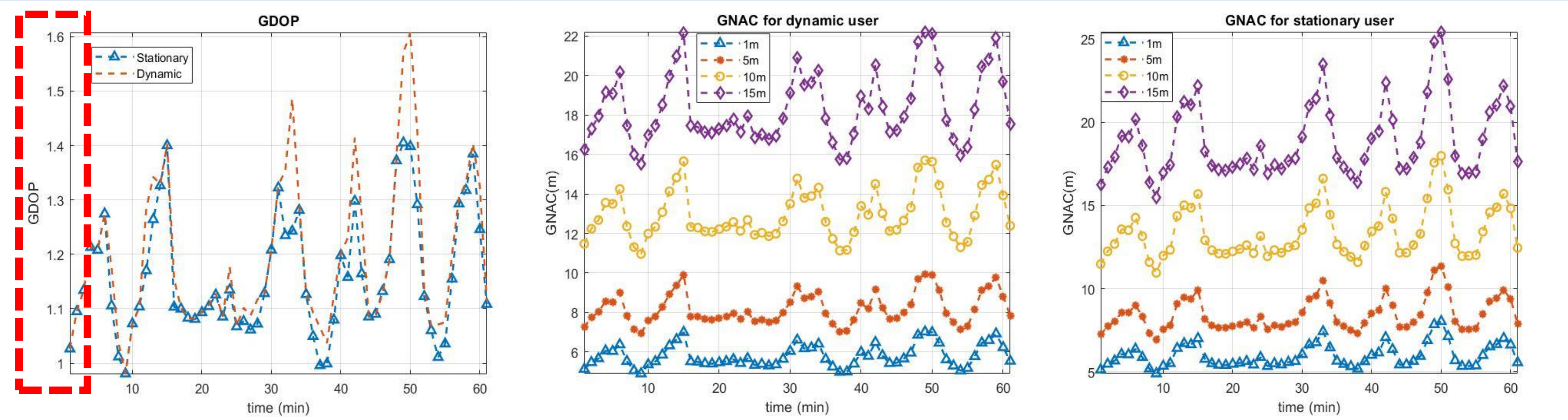
GDOP over the latitude (0 – 90 deg) North

GNAC over the latitude (0◦ - 90◦) North

Results and Analysis – III
(GDOP and GNAC for Static and Dynamic simulated users)

Case S5

Error of std 1m, 5 m, 10 m, 15 m is added



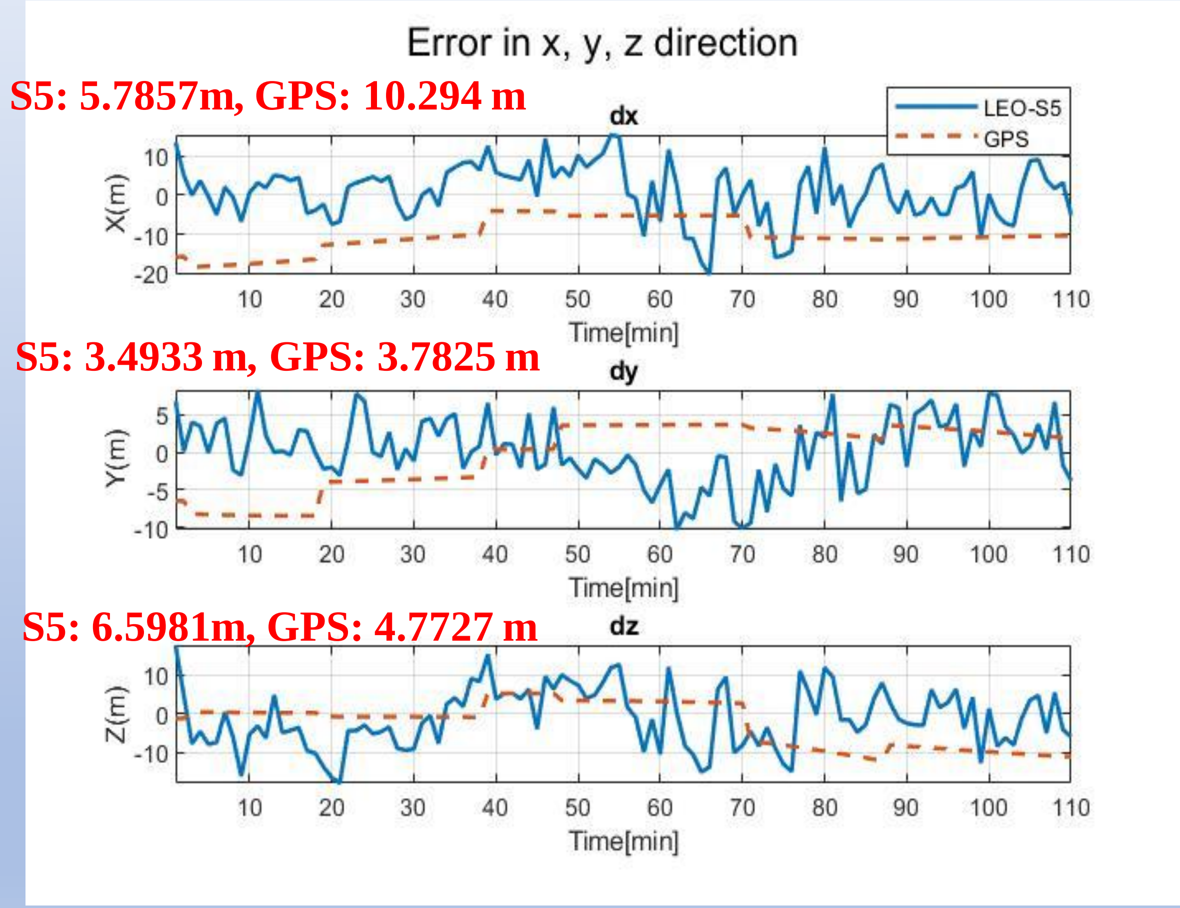
(a) GDOP for stationary and dynamics user for 60 min

(b) GNAC for stationary user for 60 min

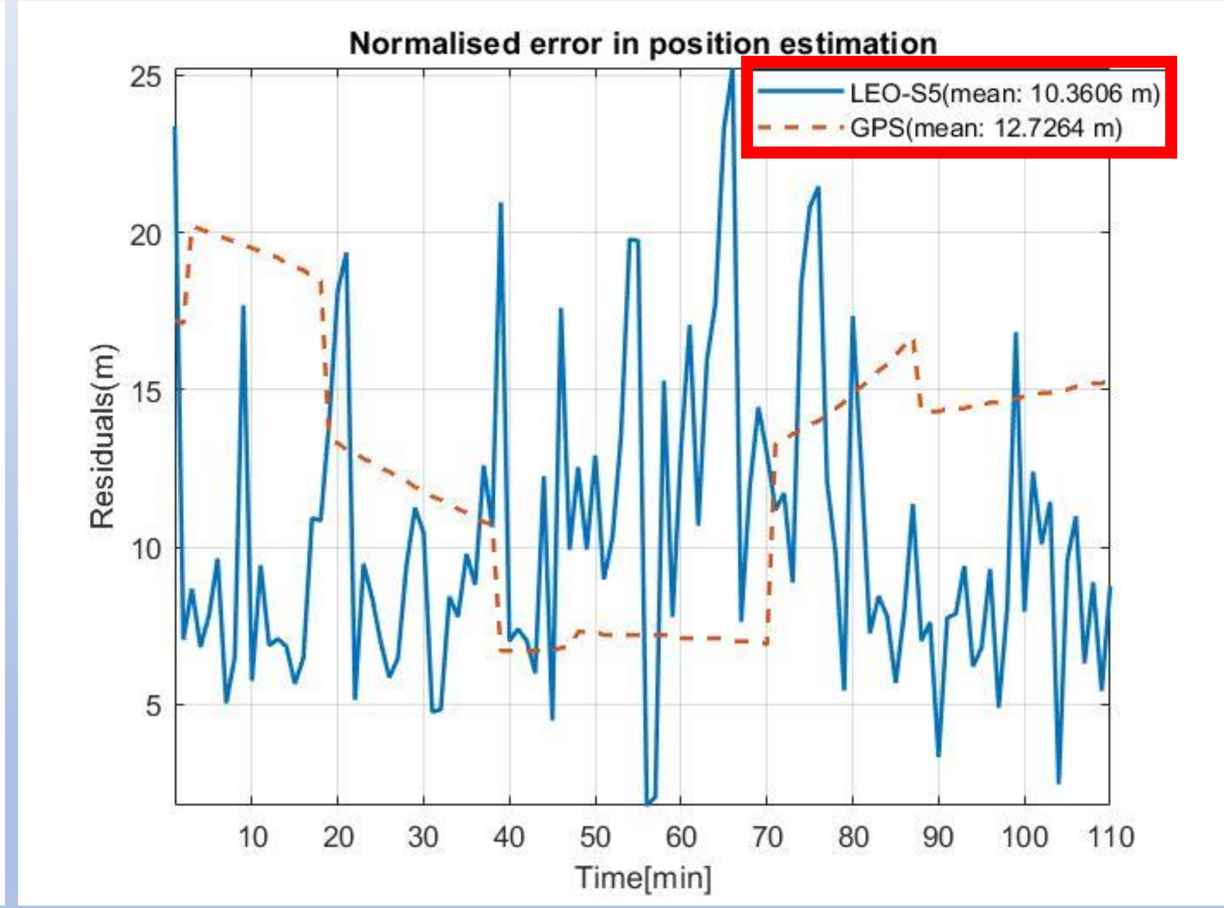
(c) GNAC for dynamic user for 60 min

Results and Analysis – IV
(LS estimation for User positioning using case S5 and GNSS)

A standard normal distribution noise of 10 m



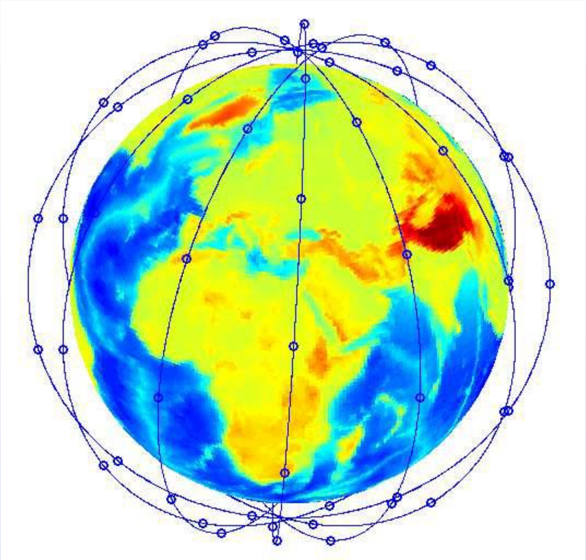
LS estimation error in x,y,z directions

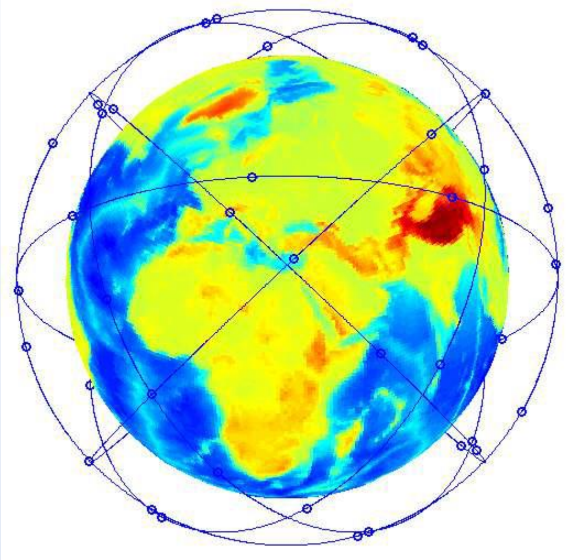


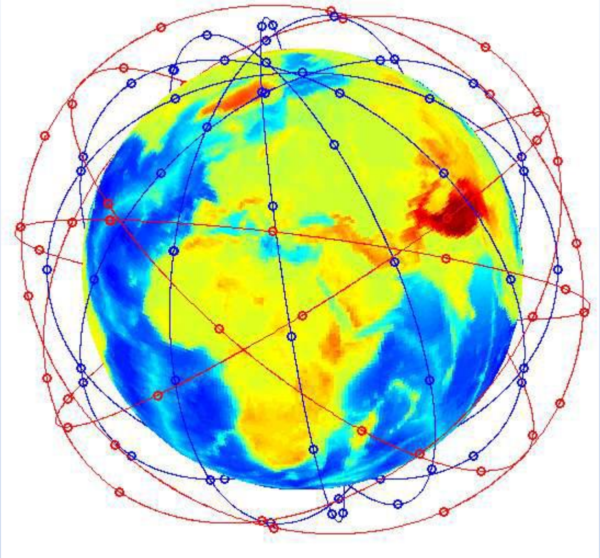
Normalised error in positioning estimation

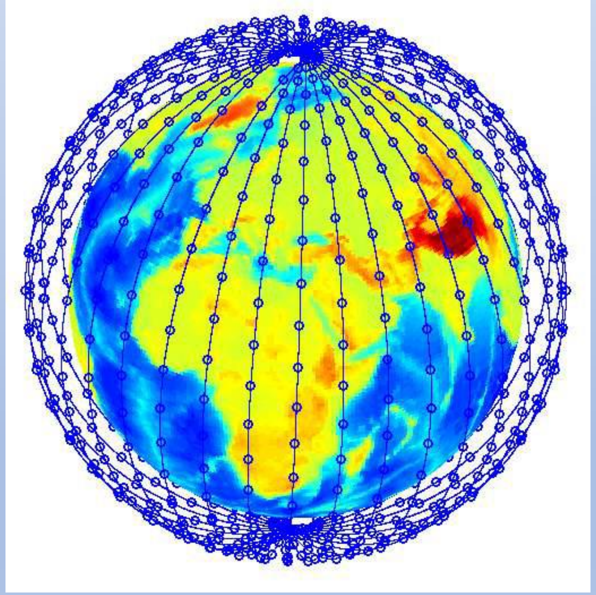
Conclusion and Future steps

- Optimised LEO based navigation can be complimentary/standalone solution in presence of threats.
- The proposed optimized configurations can provide PNT services.
- The hybrid sub-Walker configuration provides **100% global coverage with at least 5 visible satellites** at given epoch.
- The GDOP values of all obtained solutions are in the **excellent range (1 to 2).**
- Average error in positioning estimation is about **10.3606 m and 12.7264 m** for LEO and MEO **(std 10 m).**
- Improvement in DOP can relax accuracy correction models (Cheap LEO clock), and with less transmission power **(up 6 dB w.r.t. 14 dB).**
- We have plans to investigate the **integration of LEO+MEO, LEO+INS** signals in GNSS denied environment **(Urban canyon).**









Thank you !

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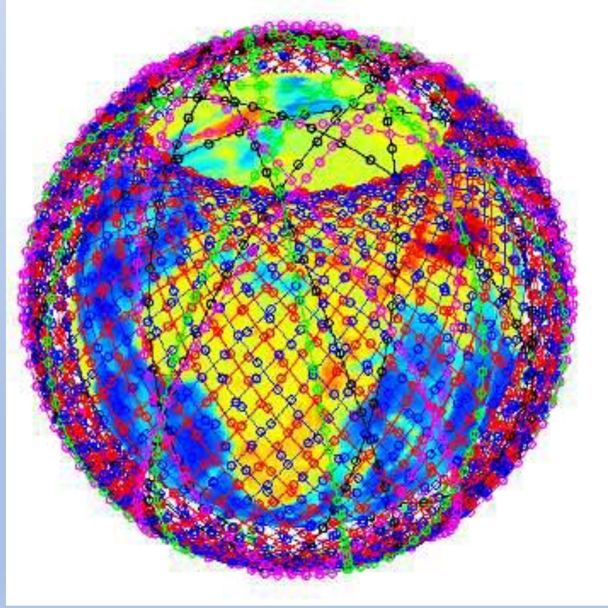

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